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Title: On exactly solvable ghost-ridden systems

Abstract:

We discuss exactly solvable systems involving integrals of motion with higher powers of momenta. If one of these integrals is chosen for the Hamiltonian, we obtain a system involving ghosts, i.e. a system whose Hamiltonian is not bounded neither from below, nor from above. However, these ghosts are benign: there is no collapse and unitarity is not violated. As an example, we consider the 3-particle Toda periodic chain, with the cubic invariant I chosen for the Hamiltonian. The classical trajectories exhibit regular oscillations, and the spectrum of the quantum Hamiltonian is discrete running from minus to plus infinity. We also discuss the classical dynamics of a perturbed system with the Hamiltonian H = I + V, where V is an oscillator potential. Such a system is not exactly solvable, but its classical trajectories exhibit not regular, but still benign behaviour without collapse. This means that also the corresponding quantum problem is well defined. The same observation can be made for exactly solvable (1+1)-dimensional field theories involving an infinite number of conservation laws: any of them can be chosen for the Hamiltonian. We illustrate this for the Sine-Gordon and KdV models. In the latter case, the Lagrangian and standard integrals of motion involve higher spatial rather than temporal derivatives. But one can always interchange x and t, after which we obtain a system with benign ghosts.